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METAL COVERING OF AIRPLANES

By J. Mathar

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METAL COVERING OF AIRPLANES*

By J. Mathar

One of the more important and much-disputed branches of airplane construction is the problem of wing covering. The multiplicity of solutions is principally due to the fact that all experience gained in machine or bridge construction is practically inapplicable to airplanes, where plate thicknesses, considered unsafe in other branches of construction, are used. In addition, no methods were available for calculating the thin plates which have a tendency to buckle even under very small loads. It was only very recently that this problem was taken up and treated in detail, notably by Professor Wagner.

We shall begin with a few remarks about the number of corrugations of a buckling and a buckled plate, and follow it with a short resumé of the principal data for an experimentally tested plate beam.

The number and shape of wrinkles or folds in smooth and corrugated plates during buckling have been treated extensively, partly theoretically and partly experimentally. We find that theory and tests agree for the smooth plate. In a poor shear test the length of the corrugation is unaffected by the plate

*"Beitrag zur Frage der Beplankung von Flugzeugen." Jahrbuch 1929 der Wissenschaftlichen Gesellschaft für Luftfahrt, pp.205-210.

thickness; in restrained plates it is 1.6 times, and for freely supported plates it is 2.7 times the width. The first top corrugation, which represents an unstable condition, has two semi-corrugations in the direction of the width. The length of the corrugation equals the width.

But in most practical cases, particularly in thinner plates, we find the number of wrinkles or folds widely at variance with the just cited theoretical figures. The reasons for this disparity lie in the inaccuracy caused by edge influences such as riveted joints, and in the rise or existence of additional stresses in the direction of the folds. The basic or bottom corrugation, with a length of 1.6 times the width, can be retained only in the exact test, where no initial deformations exist, and where additional stresses in the direction of the folds neither exist nor come into being; otherwise there will be more corrugations.

This increase in corrugations in an originally smooth thin plate occurs as follows:

Les S (Fig. 1) be the plate rigidly restrained between two strips, and P the stresses acting on it. Now when the two edges are not supported, the plate under critical shear forms corrugations, the spacing of which equals the theoretical, equivalent to 1.6 times the strip distance. This number of corrugations remains during any further stress increase or displacement.

If both edges are supported so that their spacing remains constant during the entire test, the number of corrugations, under buckling, will be the same as before. But, if we increase the shifting and, through it, the height of the corrugations (Fig. 1), we produce, due to the plate curvature, a force perpendicular to the corrugations, which seeks to reduce the size of the curvature and tends to change stage 1 through 2 and 3 into stage 4.

It will be seen from Figure 1 that the number of corrugations must treble under the respective second critical load which, however, never occurs exactly in practice, because the incipient first corrugations never agree so closely that the reversal occurs at once. If the first corrugations do not agree, they flatten out until one begins to dissolve into two, influencing those nearby and reducing the entire curvature somewhat, until equilibrium is reestablished.

The increase in corrugations is illustrated in Figures 2-5, on a rubber model test for the previously described load case, i.e., constant strip spacing. We see how the corrugations increase as the shifting increases.

If there is an incipient additional stress at right angles to P (Fig. 2) or, if during the test, higher stresses occur in the direction of the folds than in the shifting test with parallel restraining strips, the corrugations assume much higher values than in the described test.

The change in number of corrugations with respect to the incipient additional stress perpendicular to P was likewise examined with a rubber model. The test specimen for the individual added loadings is shown in Figures 6-9, while the effect of the added loading on the number of corrugations is seen in Figure 10. This latter figure (10) shows that the corrugations check with Southwell's calculated figures for $p = 0$ (Fig. 2). As the additional stress raises, the corrugations increase in number as shown by the curve.

Since it becomes evident that the second critical load is so much lower in thinner plates, the length of corrugation in a plate with low stiffness in bending becomes, in simple shear, 1.6 times the width at buckling, although this is considerably less by constant distance of the strips and, particularly when the distance increases. If the bending stiffness of the plate is zero it becomes infinitely small.

A glance over the entire structural method of plates shows that a large portion of the problems reverts to the behavior of a beam, comprising continuous struts, thin webs and vertical members or uprights, such as the walls of a metal fuselage, skin-stressed wing covering, etc.

The dimensions of this beam were in harmony with the majority of similar structural components, such as fuselage walls, etc., that is, structural members, in which the individual components, particularly the uprights, are much larger than is economical for a common girder.

The chosen dimensions of the beam may be seen in Figure 11. The cross section of the struts and uprights is the same, the plate is 0.2 mm thick.

The experiments were made within a load limit which left no appreciable permanent form changes. The measuring instruments were principally a Zeiss metering device and a Huggenberger tensiometer. The end loading was applied individually.

The objects of the test were as follows:

a) What is the effect of the number of uprights on the stiffness of a plate wall beam?

A comparison of various sheet-metal constructions shows that the ratio of beam height to panel width is far below 1. This of course is contrary to good practice, although any information on this effect should prove of interest to the designer.

b) What is the effect of riveting the plate to the uprights on the beam stiffness?

c) What is the effect of gusset plates between uprights and struts? Are they redundant or is the ratio of increased stiffness to increase in weight abnormal?

d) What is the actual stress distribution in the plate, in the uprights, and in the struts for the beam in question?

To answer these questions, we made the following experiments:

1) The plate was not fastened to the uprights, allowing the ensuing corrugations to continue undisturbed. The uprights were pin-joined to the struts. The tests were made with 1, 3, and 7 uprights. The experiments show an absence of proportionality between load and deflection, when the spacing of the uprights exceeds the beam height. (Compare the load deflection curves for 0, 1, 3, and 7 uprights in Figure 12.) We note that the load-deflection curves are almost identical for minimum loads. This is due to the fact that on the one hand a great portion of the plate does not buckle under such low loads, and on the other hand, that the ensuing tension folds or pleats in the buckling zone have originally the same direction. By increasing the load the tension pleat then suddenly becomes oblique, the load-deflection curve jumps and the stiffness of the beam is lowered.- (line a, points 1 and 2; line b, point 1).

The tension diagonals (Figs. 13-17) run from panel point to panel point, until the spacing of the uprights equals the strut height. Increasing the spacing of the uprights still further, they assume a 45-degree slope.

The end deflections of the various uprights have been reproduced in Figure 18, for a 60-kilogram constant load. Instead of assuming the value of the load-deflection curve at 60 kg as deflection factor, we extend the last linear portion of the curve to zero load and use the thus produced deflection factor. By this method we take into account that the employed loading becomes insignificant with respect to the customary loading otherwise applied. It was difficult to apply much higher loads on account of the unreliability in the instrument readings. And, as we expected, the deflection shows a sudden initial drop with respect to the number of uprights; then approaches a constant value.

2) The plate is not fastened to the uprights, but the latter are attached to the struts by gusset plates. The tests are repeated with 1, 3, and 7 uprights. It is found that the deflections naturally assume a similar behavior, depending on the number of struts, but the figures are much lower (Fig. 18).

3) The plate is fastened rigidly to the uprights and the uprights are pin-joined to the struts. The measurements are similar to 1) and 2). The principal result is that the deflections differ only slightly from those of case 2, but considerably from those of case 1 (Fig. 18). The corrugations, of course, are now interrupted at the uprights. The latter become S-shaped under deformation; the corner stiffness is ensured by the web stiffener. The measurements are similar to 1) and 2).

4) The plate is attached to the uprights and the latter are attached to the strut by gussets. The measurement of the end deflections yields about 10% lower values than in case 3 (Fig. 18). In contrast to case 3 the gusset plates raised the

weight 12%. In connection herewith we determined the stresses and deformations in a beam panel as well as the stress over all struts. The position of the measured panels a, b, c, and d is seen in Figure 11. The obtained test figures are shown at an enlarged scale in Figure 19.

Upon closer examination of the panel deformations in Figure 19, we find: The top strut is stretched, and its mean stress over b, c is 120 kg/cm² on the C.G. line of the angles. The stress is naturally lower near the four corners, because the gusset plates increase the cross section. In the center of both gusset plates the stress was found to be 136 kg/cm² while, according to Professor Wagner, it amounted to 120 kg/cm². The stress in the compression strut was 147 kg/cm² in the center of both intersection points, as measured on the C.G. line of the angles. The calculated compression stress is 156 kg/cm², according to Professor Wagner's report. Upper and lower struts being rigidly connected to the upright members by the angle plates, the deflection curves from corner to corner are S-shaped. The upright members are under 18 kg/cm² compression in contrast to 22.5 kg/cm², according to the calculation mentioned.

Inasmuch as it was impossible to place the measuring instruments other than on the struts and vertical members, we obtained a mean reduced stress in the plate, whose value is given at the four corners with respect to the angle position. The maximum stress occurs at point 6 for this panel and amounts to 172 kg/cm² at $\varphi = 36^\circ$. The stress, parallel to the struts is lower, although it can become maximum in the panel of the maximum moment. The computed value of the plate stress, according to the report mentioned, is 180 kg/cm².

The principal result of the defined stresses in the tension and compression strut prove that the measured stresses differ less than 10% from the computed stresses of Professor Wagner.

Then for comparison, we compared the deflections of the beam with those of a lattice beam obtained by cutting out the web plate. The deflections of course are incomparably higher, as, for instance, a plate wall girder with 7 vertical members is 6 times more rigid than a beam from which the plate has been removed.

In conclusion, we compared the deflections and stresses of the plate wall girder with those of a lattice beam. To be sure, we used the same strut cross sections in both. The proportions of the vertical members and of the diagonals set at 45°, were made with the intention of ensuring simultaneous weight in plate and lattice beam. Thus the diagonals and uprights had the same cross sections as the struts. Of course, these proportions do not agree with rational structural methods, but may be resorted to for comparing similarly constructed plate wall beams.

The experiment shows briefly then, that the deflection of the lattice beam is about 15% less than that of the most rigid plate wall beam, and that the difference in strut tension in the plate wall beam amounts to less than 5% of that in the lattice beam.

D i s c u s s i o n

Professor H. Wagner: Dr. Mathar's report was enjoyable for two reasons: one, because it indicates the importance which the Aachen Institute lays on problems of stresses in buckled plates, and again, because the test data, particularly the behavior of stresses, agree pretty closely with my theoretical deliberations. However, I wish to make a few remarks about certain salient points. I noticed that the loading of the plate wall in these tests was always very low in comparison to its strength. And inasmuch as the limit case of the field of tension diagonal agrees so much more closely with actual conditions, as the stress becomes higher, one might suppose that the discrepancies between the calculated and the measured stiffness, as established by Dr. Mathar, would be still lower under higher loading.

One diagram of Dr. Mathar shows the effect of the spacing and the type of upright members on the beam stiffness. There the stiffness has a tendency to reach a limit value very quickly if the vertical members are not spaced too far, in which case the stiffness becomes practically unaffected by the spacing. But there is one type (vertical members without gusset plate and not connected to plate wall), in which the stiffness shows a sudden drop. I am unable to give an explanation for it. Is this not due to some oversight in the tests? This point needs some explanation for it is important in problems dealing with the effect of neglecting the bending stiffness in a buckled plate.

Dr. Mathar stated that the number of folds at the beginning of buckling is in approximate agreement with the data of the buckling theory for plates in shear. He meant that by increasing stress, each fold should really form three new folds, while according to his experiments there were only two. Both of these statements are in contrast to my viewpoint and my test experiences.

In the narrow and long panel the pleats or folds decrease steadily as the stress increases, and the number of folds increases in like manner (for example, from 9 to 10, then to 11, etc.). I intend to publish some test data on such tension diagonal panels in the very near future, when I shall treat this question in detail. For the present, I merely wish to point to some calculations on stress and deformations in buckled plates with reference to bending stiffness in the plate, which reveal the continual effect of the stress on the width of the folds.

Professor Reissner having just asked me how I made this calculation, I shall repeat it here briefly.

As given quantities I assume the following deformations at the edges of the plate panel:

1) The mutual displacement of both struts in strut direction (due to shear) or, to be more exact, this measure of displacement divided by the strut spacing; the angle of displacement γ;

2) The elongation of the struts ϵ_x;

3) The changes in strut spacing, divided by strut spacing (ϵ_y).

Each stage of deformation thus characterized by γ, ϵ_x and ϵ_y has one definite shape of fold formation for a given plate thickness. To compute these I introduce as unknowns the fold width b, the angle of direction of the folds α, their maximum depth t, in the beam center, and several parameters a_1, a_2..., which characterize the behavior of the maximum depth of the folds along the beam height (for example, the relation of the coefficients of the Fourier Series, by which this behavior can be presented.

Now we can show the deformation of the beam with reference to the given and the unknown quantities. This deformation consists of two stages, that of the stress in bending and that of the mean longitudinal and shear stresses, respectively, so that $A = A_b + A_m = A_b\ (\gamma,\ \epsilon_x,\ \epsilon_y;\ \ b,\ \alpha,\ t,\ a_1,\ a_2) +$ $A_m\ (\gamma,\ \epsilon_x,\ \epsilon_y;\ \ b,\ \alpha,\ t,\ a_1,\ a_2)$.

Now, according to the theory of least work we have such a stage of deformation by given γ, ϵ_x, and ϵ_y, so that with respect to a variation in b, α, a_1, a_2 .... the work of form change A becomes minimum. Hence we differentiate $A = A_b + A_m$ according to each variable and obtain, when making this differential equal zero, as many equations as we have unknown variables. From these equations we then compute the unknown quantities which characterize the form of the folds.

I did not publish this calculation because I did not believe it accurate enough for my own purposes. It is not an easy matter to represent the mean plate stresses with respect to b, α, t, a_1, a_2 .... that is, to compute A_m. Furthermore, the equations are not linear with respect to the unknown quantities, so that in order to avoid all too complicated solutions even the type of unknown quantities is subject to certain limitations.

Since, however, A_b and A_m are wholly dependent on the width of the folds b, this width must steadily decrease as the stress continues to increase (for example, by steady in crease in γ).

<u>Dr. Mathar</u>: Anent Professor Wagner's explanations, I wish to remark: I do not believe that curve 1 in Figure 18 will approach curve 2 under increasing stress, as long as we remain within the elastic limit. Even under a stress five times higher

than the present, the deflections (7 vertical members) are still proportional to the stress, so the relation of curve 1 and curve 2 remain the same. Moreover, under the present load of 60 kilograms the plate bulges at nearly every point, so the difference in the two curves can become only imperceptibly less as the stress increases. Regarding the difference of curves 1 and 2 with 3 and 4 in Figure 18, I wish to state that the difference between curves 1 and 2 hinges above all on the gusset plates, which are followed by a distortion of the uprights and an added waviness of the struts.

A slight effect on the beam stiffness is found when using gusset plates, in so far as they prevent sagging of the struts between two verticals, better than in case 1. But in case 3 the plate is fastened to the uprights and the deformations of the latter resemble those of case 2, hence the addition of gusset plates (case 4) leads us to expect less increase in stiffness than in case 1 compared to case 2. As to the increase in corrugations in a buckled plate - a problem which cannot be solved with linear differential equations - I again repeat that this triplication is feasible and must occur only in the exact case where the corrugations agree completely. But in all practical cases where, of course, this agreement is lacking, we always will find a steady increase in corrugations, as was illustrated in Figure 10. With these last cases we must likewise class Professor Wagner's case of two free edges, where a new corrugation begins to form which affects all others with it.

Translation by J. Vanier,
National Advisory Committee
for Aeronautics.

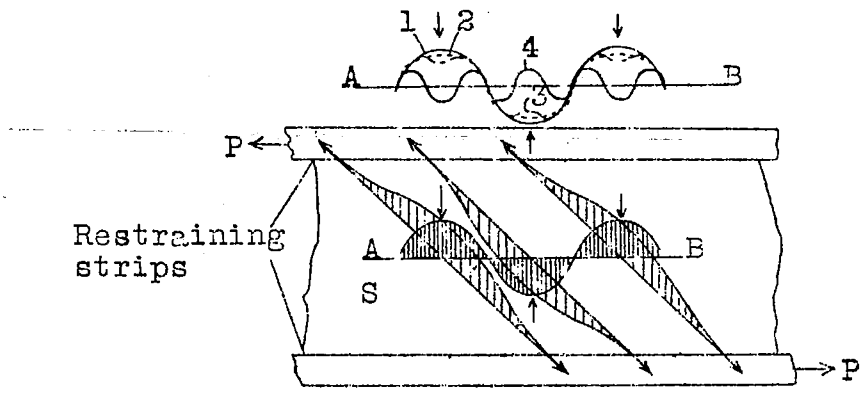

Fig.1 Diagram showing increase in corrugations.

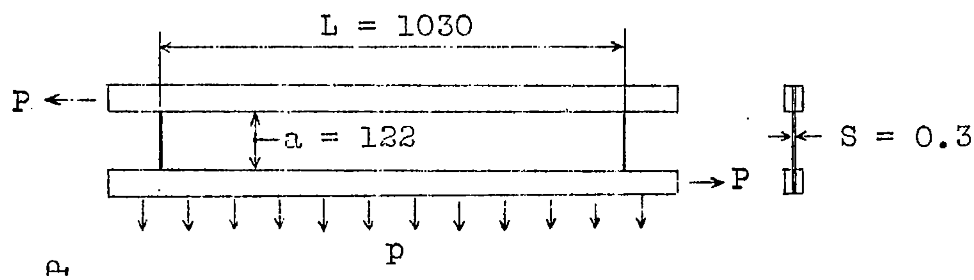

Fig.2

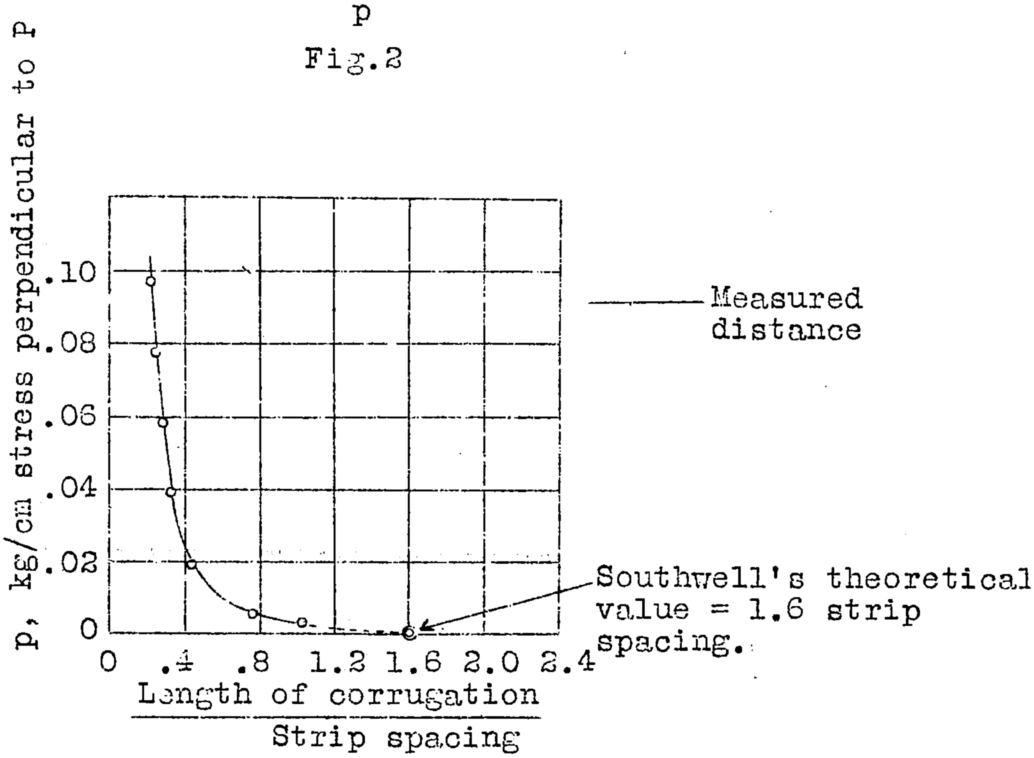

Fig.10

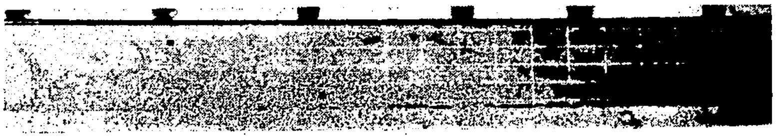

Fig.3

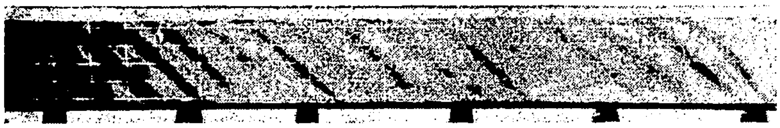

Fig.4

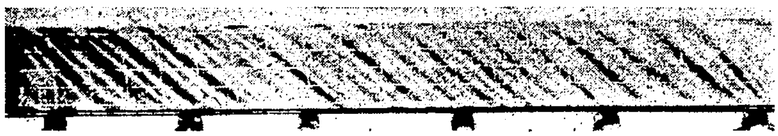

Fig.5

Figs.3-5 Increase in corrugation by increasing displacement. (Constant edge spacing.)

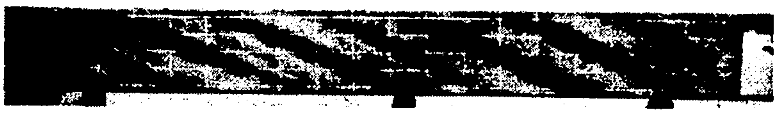

Fig.6 p - 0.0025 kg/cm

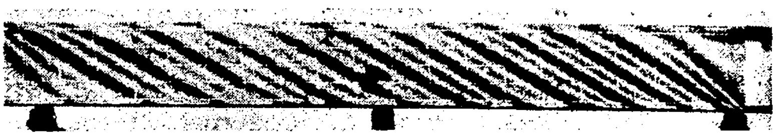

Fig.7 p - 0.004 kg/cm

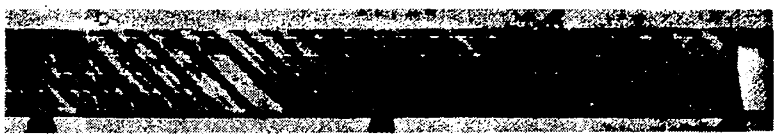

Fig.8 p - 0.01 kg/cm

Fig.8 p - 0.08 kg/cm

Figs.6-9 Number of corrugations with respect to added stress p.

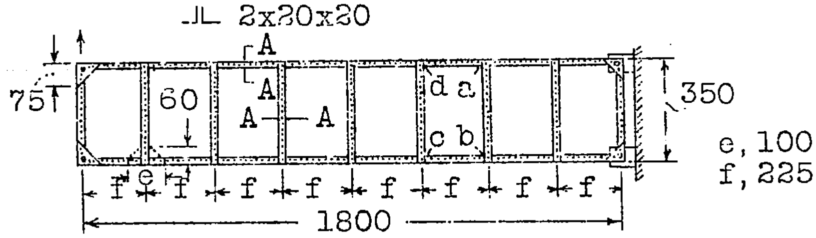

Fig. 11 The tested plate wall beam

a, Frame without uprights. (Fig. 14)
b, Frame with 1 upright. (Fig. 15)
c, Frame with 3 upright. (Fig. 16)
d, Frame with 7 upright. (Fig. 17)

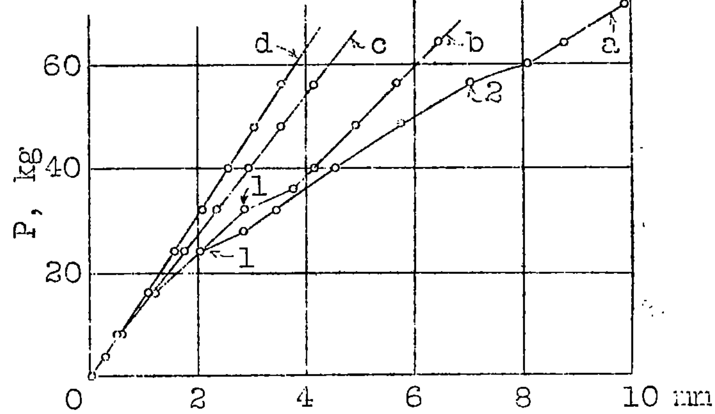

Fig. 12 Deflection of beam end plotted, for 0, against stress 0 (curve a) for 1 (curve b) for 3 (curve c) for 7 (curve d) vertical members

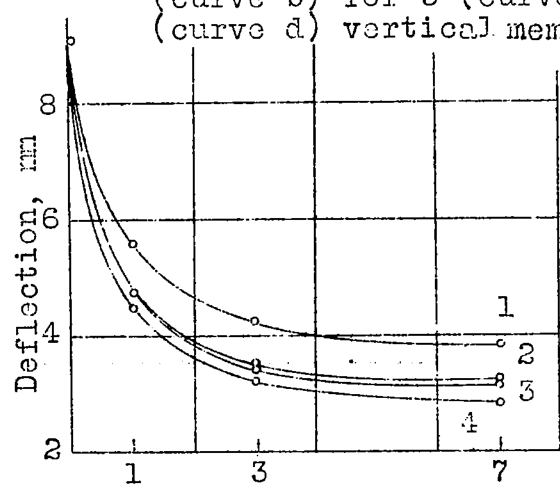

Fig. 18 Deflections of beam end for 0,1,3 and 7 uprights.(P=60 kg)

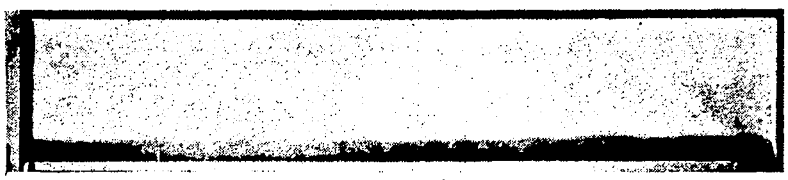

Fig.13

Fig.14

Fig.15

Fig,16

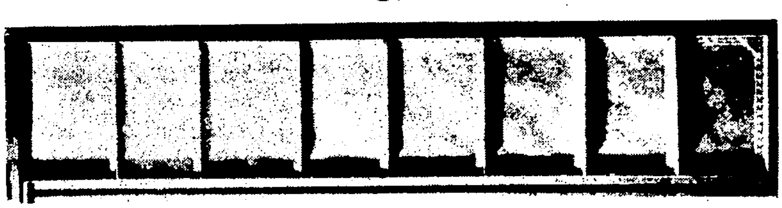

Fig.17

Figs.13-17 Direction of tension folds in web plate for 0,1,3 and 7 uprights.

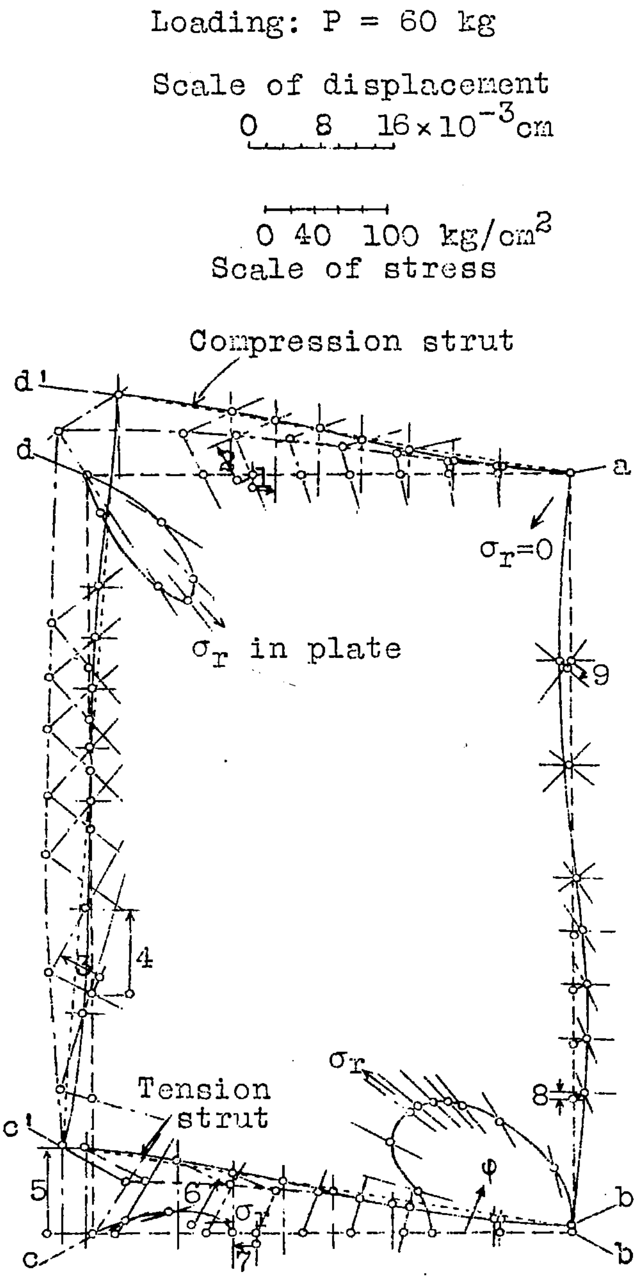

Fig.19 Stresses and displacements in examiner beam panel a,b,c, & d (Fig.11).